## Thinking like a Physicist: Predicting the Size of Atoms

In this exercise you will use *dimensional analysis* to gain insights into the nature of atoms. Just as the fundamental constant of nature, (the speed of light), is important in any problems involving relativity theory, the fundamental constant of nature, (Planck’s constant), is important in describing any system that is quantum mechanical in nature, like the atom.

Part 01: Estimate the Size of Atoms

Using Dimensional Analysis

The idea of an “atom” as an indivisible unit of matter was conceived independently by ancient Indian and Greek philosophers thousands of years ago. However, it wasn’t until physicists in the early 20th century realized the quantum nature of our universe that any real progress was made in understanding what an atom is and how it works.

One of the simplest properties of atoms that we might wish to know is their size. Rather than merely *measuring* it, can we actually *predict* this size, based on our understanding of the laws of nature? In other words, can we actually find a formula that says: “Size of atom = X”, where X is some combination of *fundamental constants of nature*?

Thinking like a physicist you might reason that the size of the simplest atom (hydrogen) must depend on:

1. Some fundamental constant(s) of nature characterizing *what the atom is made of*. A hydrogen atom has one electron and one proton. The magnitude of electric charge on each is  
    C, and this should be important. (For example, increasing would increase the force of electrostatic attraction between the two, and hence decrease the size of the atom.) The mass of the electron, kg, should also be important. Why? If is increased, do you think the size of the atom will increase or decrease? Note: As long as the mass of the proton is much greater than the mass of the electron (which it is), why is its mass *not* important?
2. Some fundamental constant of nature characterizing *the forces at work inside the atom*, i.e., the strength of the electrostatic force pulling the electron toward the nucleus. After all, everything else being equal, the stronger this force is, the smaller the atom will be. This force is , where N m2/C2 is the electrostatic force constant. In this formula, we’ve already accounted for the electric charge () in #1 above. And we ignore the radius (it’s a variable, not a fundamental constant of nature). That leaves as the fundamental constant of nature characterizing the strength of the attractive force between charged particles in the atom.
3. Some fundamental constant of nature reflecting the fact that *quantum mechanics is important to how atoms work*, i.e., Planck’s constant, J s. (It turns out that the “reduced Planck’s constant”, , pronounced “h-bar”, is often more useful than .) This was the key fundamental constant of nature missing until the early 20th century.

So thinking like a physicist, you might reason that the size of a hydrogen atom must depend on the following four fundamental constants of nature: , , , and . Can you put these together in a combination with the *dimensions of length* (i.e., units of metres), and hence guess a formula for the size of the hydrogen atom? Substitute numbers into your formula. Does your answer seem correct?

Part 02: Estimate the Size of Atoms Using

the Wave Nature of Particles

Imagine that the electron moves with a speed around the nucleus of the atom in a circular orbit of radius .

1. Start with Newton’s law, , and substitute the electrostatic force for , and the centripetal acceleration for ;
2. Substitute for , where is the electron momentum in its circular orbit;
3. Use the de Broglie relation to express in terms of the electron wavelength.

The hydrogen atom has different sizes depending on its energy. In a high energy state, the electron is far from the nucleus and the atom is larger; in a low energy state, the electron is closer to the nucleus and the atom is smaller. Let’s focus our attention on the lowest energy state, or *ground state*, in which the atom has the smallest size, i.e., the electron is “orbiting” in the smallest possible orbit.

The kinetic energy of the electron can be expresses as , so if we use the de Broglie relation we see that the energy of the electron is inversely related to its wavelength. Larger wavelength means smaller energy. Imagining that the “electron wave” is circulating around the nucleus in a circle of radius , what is the *maximum* wavelength this wave could have? Substitute this maximum wavelength for the in the formula you derived above, and solve the resulting equation for . Express you answer in terms of () and compare with your answer in Part 01. Do they agree?

Estimating the size of atoms in terms of fundamental physical constants of nature was one of the greatest early achievements of quantum mechanics. Which way of estimating the size of the atom was easier, Part 01 or Part 02? Observe the simplicity and power of dimensional analysis!